E

$$y'' + x^2 y = 0$$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y'' = \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n$$

$$y'' + x^2 y = \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n + x^2 \sum_{n=0}^{\infty} a_n x^n$$

$$= \quad '' \quad + \sum_{n=0}^{\infty} a_n x^{n+2}$$

$$= \quad '' \quad + \sum_{n=2}^{\infty} a_{n-2} x^n$$

$$= 2 \cdot 1 \cdot a_2 x^0 + 3 \cdot 2 \cdot a_3 x^1 + \sum_{n=2}^{\infty} \left((n+2)(n+1) a_{n+2} + a_{n-2} \right) x^n = 0$$

For $n \geq 2$, $(n+2)(n+1) a_{n+2} + a_{n-2} = 0$

$$2a_2 = 0 \qquad 6a_3 x = 0x \qquad a_{n+2} = -\frac{1}{(n+2)(n+1)} a_{n-2}$$

$$a_2 = a_3 = 0$$

LET $a_0 = 1,\ a_1 = 0$

$$n \geq 2 \rightarrow a_{n+2} = -\frac{1}{(n+2)(n+1)} a_{n-2}$$

$a_2 = a_3 = 0$

$$n=2:\ a_4 = -\frac{1}{4 \cdot 3} a_0 = -\frac{1}{4 \cdot 3}$$

$$n=3;\ a_5 = \underbrace{-\frac{1}{5 \cdot 4}}_{\text{UNNECESSARY}} \underbrace{a_1}_{=0} = 0 = a_9 = a_{13} = a_{17} = \cdots$$

$$n=4:\ a_6 = \quad a_2 = 0 = a_{10} = a_{14} = a_{18} = \cdots$$

$$n=5:\ a_7 = \quad a_3 = 0 = a_{11} = a_{15} = a_{19} = \cdots$$

$$n=6:\ a_8 = -\frac{1}{8 \cdot 7} a_4 = -\frac{1}{8 \cdot 7} \cdot -\frac{1}{4 \cdot 3} = \frac{1}{8 \cdot 7 \cdot 4 \cdot 3}$$

$$n=10:\ a_{12} = -\frac{1}{12 \cdot 11} a_8 = -\frac{1}{12 \cdot 11 \cdot 8 \cdot 7 \cdot 4 \cdot 3}$$

$$y_1 = \underset{n=0}{1} - \underset{n=1}{\frac{1}{4 \cdot 3} x^4} + \underset{n=2}{\frac{1}{8 \cdot 7 \cdot 4 \cdot 3} x^8} - \underset{n=3}{\frac{1}{12 \cdot 11 \cdot 8 \cdot 7 \cdot 4 \cdot 3} x^{12}} + \cdots$$

(IF YOU WANTED THE FIRST 4 NON-ZERO TERMS)

$$= 1 + \sum_{n=1}^{\infty} (-1)^n \frac{1}{3 \cdot 4 \cdot 7 \cdot 8 \cdot 11 \cdot 12 \cdots (4n-1)(4n)} x^{4n}$$

(IF YOU WANT THE ENTIRE FUNCTION y_1.)

LET $a_0=0,\ a_1=1$

$a_2=a_3=0$

$$n \geq 2 \rightarrow a_{n+2} = -\frac{1}{(n+2)(n+1)}\,a_{n-2}$$

$a_0=0=a_4=a_8=a_{12}\cdots$

$a_2=0=a_6=a_{10}=a_{14}\cdots$

$a_3=0=a_7=a_{11}=a_{15}\cdots$

$n=3:\ a_5 = -\frac{1}{5\cdot 4}\,a_1 = -\frac{1}{5\cdot 4}$

$n=7:\ a_9 = -\frac{1}{9\cdot 8}\,a_5 = \frac{1}{9\cdot 8\cdot 5\cdot 4}$

$n=11:\ a_{13} = -\frac{1}{13\cdot 12}\,a_9 = -\frac{1}{13\cdot 12\cdot 9\cdot 8\cdot 5\cdot 4}$

$$y_2 = \underset{n=0}{x} - \underset{n=1}{\frac{1}{5\cdot 4}x^5} + \underset{n=2}{\frac{1}{9\cdot 8\cdot 5\cdot 4}x^9} - \underset{n=3}{\frac{1}{13\cdot 12\cdot 9\cdot 8\cdot 5\cdot 4}x^{13}} + \cdots$$

$$= x + \sum_{n=1}^{\infty} (-1)^n \frac{1}{4\cdot 5\cdot 8\cdot 9\cdot 12\cdot 13\cdots(4n)(4n+1)}\,x^{1+4n}$$

$$y = c_1 y_1 + c_2 y_2$$

LVE $(x^2+1)y'' - xy' + y = 0$ USING SERIES.

$\hookrightarrow y'' \underbrace{- \frac{x}{x^2+1}}_{p} y' + \underbrace{\frac{1}{x^2+1}}_{q} y = 0$

p, q CONT ON $(-\infty, \infty)$

SO E+U SAYS THERE IS A UNIQUE SOL'N FOR EVERY IVP FROM THIS DE

$\sum_{n=0}^{\infty} a_n x^n$

$\sum_{n=1}^{\infty} n a_n x^{n-1}$

$= \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$

$$+1) \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - x \sum_{n=1}^{\infty} n a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=1}^{\infty} n a_n x^n + \sum_{n=0}^{\infty} a_n x^n$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^n + \sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n - \sum_{n=1}^{\infty} n a_n x^n + \sum_{n=0}^{\infty} a_n x^n$$

$$2 \cdot 1 \cdot a_2 x^0 + 3 \cdot 2 \cdot a_3 x^1 - 1 \cdot a_1 x^1 + a_0 x^0 + a_1 x^1$$

$$+ \sum_{n=2}^{\infty} \left[n(n-1) a_n + (n+2)(n+1) a_{n+2} - n a_n + a_n \right] x^n = 0$$

$(2a_2+a_0) + (6a_3 \cancel{-a_1} + a_1)x$

$$+\sum_{n=2}^{\infty}\left[(n+2)(n+1)a_{n+2}+(\underbrace{n^2-n-n+1}_{\substack{n^2-2n+1\\=(n-1)^2}})a_n\right]x^n=0$$

$2a_2+a_0=0 \longrightarrow a_2=-\frac{1}{2}a_0$

$6a_3x=0x \longrightarrow a_3=0$

$(n+2)(n+1)a_{n+2}+(n-1)^2a_n=0$

$$a_{n+2}=-\frac{(n-1)^2}{(n+2)(n+1)}a_n \quad \text{FOR } n\geq 2$$

RECURRENCE RELATION
FOR COEFFICIENTS

LET $a_0=1, a_1=0$

$\hookrightarrow a_1=0=a_3=a_5=a_7=\cdots$

$a_2=-\frac{1}{2}a_0=-\frac{1}{2}$

$=2: a_4=-\frac{1^2}{4\cdot 3}a_2=\frac{1^2}{4\cdot 3\cdot 2}$ $\xleftarrow{n=1}$ $\xleftarrow{n=2}$

$$a_{2n}=(-1)^n\frac{1^2\cdot 3^2\cdot 5^2\cdots(2n-3)^2}{(2n)!}$$

$=4: a_6=-\frac{3^2}{6\cdot 5}a_4=-\frac{3^2\cdot 1^2}{6\cdot 5\cdot 4\cdot 3\cdot 2}$ $\xleftarrow{n=3}$

CAN BE SIMPLIFIED

$=6: a_8=-\frac{5^2}{8\cdot 7}a_6=\frac{5^2\cdot 3^2\cdot 1^2}{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}$ $\xleftarrow{n=4}$